Title: Melnikov-type method for splitting of separatrices for an explicit range of small parameter

Author: Piotr Zgliczynski (Jagiellonian University, Krakow, Poland), Maciej Capiński (AGH, Krakow, Poland)

One of the main tools to determine the existence of (or non-existence of) chaos in a perturbed hamiltonian system is the Melnikov theory. In this theory, the distance between the stable and unstable manifolds of the perturbed system is calculated up to the first order term, hence the precise range of small parameter for which the transversal intersection exists is unknown.

We propose the method which allows to compute an explicit range of small parameter for which the intersection exists, with the aim to obtain the size of the parameter from which the continuation with other direct geometric tools is possible. The method is computer assisted (a computer assisted proof). We applied it to the system

$$(x', y') = \left(y - \varepsilon \cos(t) y^2, x - x^2\right), \tag{1}$$

given by the following Hamiltonian

$$H_\varepsilon(x, y) = \frac{y^2 - x^2}{2} + \frac{x^3}{3} - \frac{\varepsilon y^3 \cos t}{3}. \tag{2}$$

We have proved the existence of transversal intersection for $\epsilon \in (0, 10^{-3}]$. In this example we also checked that for $\epsilon = 10^{-3}$ the 'direct' method of establishing transversal intersection also works, so these computations can be continued to much larger values of ϵ.

To compute the Melnikov distance our method combines two ingredients, both computer assisted

- geometric method to establish explicit bounds for normally hyperbolic invariant manifolds (NHIM) and their stable and unstable fibers, together with their dependence of parameter. The NIHM in question in our example the periodic orbits $(0, 0, t)$.
- the rigorous C^2-integration of our system away from the NHIM.

The method can be generalized to many dimensions.