Minkowski's question mark function and measure.

Giorgio Mantica, Universita' dell'Insubria

Minkowski's famous question mark function appears frequently in dynamical systems of number--theoretical origin: it makes the isomorphism between the tent and the Farey map, it is the coding function of motions in billiards in hyperbolic space, et cetera. In particular, it is the distribution function of a singular continuous measure, which is invariant under a modified Farey map. This measure is also the balanced measure of an Iterated Function System composed of Moebius maps. In this talk I will show how to use this fact for the numerical calculation of this measure and for the resolution of two open problems on its analytical nature.

Firstly, because of Favard's theorem, a measure is uniquely determined by the infinite family of its orthogonal polynomials. In turn, these latter are uniquely encoded in the Jacobi matrix of their three--terms recurrence coefficients: computing this matrix is therefore a crucial step, much in the same way as sequencing the genome of a living being. In the first part of this talk I will show how this goal can be achieved by a numerical algorithm with controlled errors.

Secondly, once the Jacobi matrix has been computed, many properties of the associated measure can be ``decoded'' from it, like e.g. continuity properties. It is well known that Minkowski's measure is singular continuous; yet two of its finer characteristics, which stem from the theory of logarithmic potential in the complex plane, are still conjectural: whether the measure is regular (in the sense of Ullman, Stahl and Totik) and whether it belongs to a Nevai class. I will briefly define these concepts and I will present numerical evidence in favor of the validity of both conjectures. I will also show pictures of the atomic measures composing the Gaussian integration of Minkowski's measure, which reveal fascinating structures that can be well described by an asymptotic formula.

Time permitting, in the end I will present a few hints to the proof of the presented conjectures that are suggested by the numerical techniques.