Quasi-periodic solutions for state dependent delay equations.

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We consider delay differential equations in which the delay depends on the state of the system These equations appear naturally in electrodynamics for particles interacting with retarded potentials (the delay is proportional to the distance) as well as in several biological models. We note that for these equations the phase space is innite dimensional and not easy to describe. Questions such existence, uniqueness, dependence on parameters are still puzzling. We develop a theory of quasi-periodic solutions that bypasses the questions of existence for general initial data. We develop a functional equation for the quasiperiodic equations and study them by functional analysis methods. The main results are stated in an a-posteriori format that states that given proximate solutions that satisfy some explicit non-degeneracy solutions, there are true solutions nearby. This can be used to justify some numerical solutions that have been produced. We show that in a one-parameter family, there are smooth solutions. Furthermore, we can find a large measure set where the quasi-periodic solution is analytic. We conjecture that this is regularity is optimal. We also develop a theory of stable/unstable manifolds.

This is joint work with Xiaolong He.