Rigorous integration of dissipative PDEs revisited: chaos in the Kuramoto-Sivashinsky equations.
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In a series of articles Zgiclzynski provided an algorithm for rigorous integration of dissipative PDEs. The main idea of this approach is to transform the problem of rigorous integration of a PDE into rigorous integration of differential inclusion which is sum of a finite dimensional ODE and an infinite dimensional perturbation.

Another algorithm was proposed by Arioli and Koch and successfully applied to computer assisted verification of the existence and hyperbolicity of a periodic orbit in the Kuramoto-Sivashinsky equation.

We propose an alternative algorithm which directly solves infinite dimensional ODE. The key ingredient is an algorithm for automatic differentiation for dissipative PDEs.

As an application of the proposed algorithm we give a computer assisted proof of the existence of chaotic dynamics in the Kuramoto-Sivashinsky equation. We show that for certain parameter value a Poincare map restricted to some invariant set is semiconjugated to the shift dynamics on two symbols. Moreover, the invariant set contains countable infinity of periodic orbits.