A new perspective for rigorous integration of dissipative PDEs.

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In a series of articles Zgiczynski provided an algorithm for rigorous integration of dissipative PDEs. The main idea of this approach is to transform the problem of rigorous integration of a PDE into rigorous integration of differential inclusion which is sum of a finite dimensional ODE and an infinite dimensional perturbation.

We propose an alternative algorithm which directly solves infinite dimensional ODE. The key ingredient is an algorithm for automatic differentiation for dissipative PDEs. This approach gives also the possibility to rigorous integration of the cone field associated to this system and thus obtain C^1 information.

As an application of the algorithm we would like to give a computer assisted proof that the paradigmatic Kuramoto-Sivashinsky equation admits chaotic dynamics for a certain parameter value. At the moment I'm writing this abstract the algorithm is being implemented and tested. Therefore, we have only a partial result about the existence of chaotic dynamics in high dimensional (15,20,25) approximations to the infinite dimensional system. This result strongly suggests the presence of chaos in infinite dimensional system.