Parametric subharmonic instability of internal waves: locally confined beams versus monochromatic wavetrains

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Internal gravity wavetrains in continuously stratified fluids are generally unstable as a result of resonant triad interactions which, in the unviscid limit, amplify short-scale perturbations with frequency equal to one half of that of the underlying wave. This so-called parametric subharmonic instability (PSI) has been studied extensively for spatially and temporally monochromatic waves. We present an asymptotic analysis of PSI for time-harmonic plane waves with locally confined spatial profile, in an effort to understand how such wave beams differ, in regard to PSI, from monochromatic waves. For beams with general localized profile, it is found that triad interactions are not strong enough to bring about instability in the limited time that subharmonic perturbations overlap with the beam. On the other hand, for quasi-monochromatic wave beams whose profile comprises a sinusoidal carrier modulated by a locally confined envelope, PSI is possible if the beam is wide enough. An important exception arises when the beam frequency is nearly twice the inertial frequency due to background rotation; under this condition, PSI is possible for beams of general locally confined profile, as subharmonic perturbations of near-inertial frequency have small group velocity and stay in contact with the underlying beam longer, thus extracting more energy. The theoretical predictions are in keeping with numerical simulations and observations.