On the universal optimality of the 600-cell: the Levenshtein framework lifted (jointly with Boyvalenkov, Hardin, Saff, Stoyanova)

Peter Dragnev, Purdue University Fort Wayne

The potential energy of a spherical code $C\subset \mathbb{S}^{n-1}$ with interaction potential h is defined as $E(C,n,h):=\sum_{x\not= y\in C} h(\langle x,y\rangle)$. In 2016 the authors derived a universal lower bound on energy for absolutely monotone potentials
$E(C,n,h) \geq N^2 \sum_{i=1}^m \rho_i h(\alpha_i) $,
where the nodes $\{ \alpha_i \}$ and weights $\{ \rho_i \}$ depend only on the cardinality N of C and dimension n and are obtained from a quadrature rule framework studied by Levenshtein in relation to maximal codes. The lower bound is attained for all universally optimal codes discovered by Cohn and Kumar, but the 600-cell (a code with 120 points on $\mathbb{S}^3$).

In this talk we present a method for lifting the Levenshtein framework (increase m). As a consequence we obtain a solution of the LP problem associated with the 600-cell and obtain a characterization of the optimal polynomials of degree at most 17 as a linear combination of three extremal polynomials, one of which is the one introduced by Cohn-Kumar in their proof.

.