Fast multipole methods in arbitrary dimensions

George Biros, The University of Texas at Austin

The Fast multipole method (FMM) is ubiquitous in science and engineering. Roughly speaking, it accelerates computations with dense matrices. For example, for a square dense matrix of size N matrix vector multiplication requires $O(N^2)$ work. FMM can accelerate this operation to $O(N)$ work. FMM methods where originally defined using analytical expansion methods but over the years they have been ``algebrized'' to an increasing degree. Current state-of-the art FMM methods are only applicable to problems endowed with geometrical information, typically point coordinates. underlying geometric information (typically point coordinates).

In this talk I will present two algorithms: ASKIT and GOFMM. ASKIT (approximate skeletonization kernel independent treecode) dependents only on the local intrinsic dimension of the dataset as opposed to the dimension of the ambient space of the dataset. GOFMM generalizes the FMM method to arbitrary symmetric positive definite matrices--no geometric information is necessary. Under certain circumstances, GOFMM enables an approximate matrix-vector multiplication in O(N log N) or even O(N) time. Both ASKIT and GOFMM scheme belongs to the hierarchical matrix approximation methods. As a highlight, we will report results for Gaussian kernel matrices with 100 million points in 64 dimensions, and for eight million points in 1000 dimensions.