Hyperuniformity on the sphere

Peter Grabner, Graz University of Technology

The concept of hyperuniformity had been introduced by S.~Torquato and F.~Stillinger to measure regularity of distributions of infinite particle systems in $\mathbb{R}^d$. An infinite particle system $X\subset\mathbb{R}^d$ is called hyperuniform, if the variance (with respect to the thermodynamic limit) of the number of points in a \textbf{large} ball is smaller than ``usual'': \begin{equation*} \mathbb{V}\#(X\cap(\mathbf{x},R))=\mathcal{O}(R^{d-1})\text{ for }R\to\infty. \end{equation*}
Notice that this variance is of order R^d for Poisson point processes.

We generalise this concept to the sphere by considering sequences of \textbf{finite} point sets $(X_N)_N$ (with $\# X_N=N$). The phenomenon of a ``smaller than usual'' variance of the point counting function is then observed in three different regimes for the opening angle of spherical caps. We will discuss several examples of hyperuniform sequences of point sets.