Random walks on perfect lattices and the sphere packing problem.

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Motivated by the search for best lattice sphere packings in Euclidean spaces of large dimensions we study randomly generated perfect lattices in moderately large dimensions (up to d = 19 included). Perfect lattices are relevant to the solution of the problem of lattice sphere packing, because the best lattice packing is a perfect lattice and because they can be generated easily by an algorithm. Their number however grows super-exponentially with the dimension so to get an idea of their properties we propose to study a randomized version of the algorithm and to define a random ensemble with an effective temperature in a way reminiscent of a Montecarlo simulation. We therefore study the distribution of packing fractions and kissing numbers of these ensembles and show how as the temperature is decreased the best know packers are easily recovered. We find that, even at infinite temperature, the typical perfect lattices are considerably denser than known families (like Ad and Dd). We also find properties of the random walk which are suggestive of a glassy system already for moderately small dimensions. We also analyze local structure of the Voronoi graph of perfect lattices conjecturing that this is a scale-free network in all dimensions with constant scaling exponent 2.6 ± 0.1.