The upper density of sets avoiding norm one in the real space of dimension n

Christine Bachoc, Université de Bordeaux

What is the largest density of a subset of the real space of dimension n such that the difference of any pair of points of this subset does not achieve norm one ? This question is related to the long standing problem of the determination of the chromatic number of the plane. In this talk we will focus on the case when the unit ball is a polytope. If the polytope tiles space by translations, together with Sinai Robins we have conjectured that the answer is 1/2^n. An upper bound can be obtained from the Fourier transform of any measure supported on the boundary of the polytope. Optimizing over the measure appears to be a difficult problem; in particular, point measures lead to interesting polynomial optimization problems.

This is joint work with Philippe Moustrou and Sinai Robins.