On Approximating the Covering Radius and Finding Dense Lattice Subspaces.

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Integer programming, the problem of finding an optimal integer solution satisfying linear constraints, is one of the most fundamental problems in discrete optimization. In the first part of this talk, I will discuss the important open problem of whether there exists a single exponential time algorithm for solving a general n variable integer program, where the best current algorithm requires $n^{O(n)}$ time. I will use this to motivate a beautiful conjecture of Kannan & Lovasz (KL) regarding how ""flat"" convex bodies not containing integer points must be.

The l_2 case of KL was recently resolved in breakthrough work by Regev & Davidowitz `17, who proved a more general ""Reverse Minkowski"" theorem which gives an effective way of bounding lattice point counts inside any ball around the origin as a function of sublattice determinants. In both cases, they prove the existence of certain ""witness"" lattice subspaces in a non-constructive way that explain geometric parameters of the lattice. In this work, as my first result, I show how to make these results constructive in $2^{O(n)}$ time, i.e. which can actually find these witness subspaces, using discrete Gaussian sampling techniques. As a second main result, I show an improved complexity characterization for approximating the covering radius of a lattice, i.e. the farthest distance of any point in space to the lattice. In particular, assuming the slicing conjecture, I show that this problem is in coNP for constant approximation factor, which improves on the corresponding $O(\log^{3/2} n)$ approximation factor given by Regev & Davidowitz's proof of the l_2 KL conjecture.