Tight framelets and fast framelet filter bank transforms on manifolds

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Data in practical application with some structure can be viewed as sampled from a manifold, for instance, data on a graph and in astrophysics. A smooth and compact Riemannian manifold M, including examples of spheres, tori, cubes and graphs, is an important geometric struncture. In this work, we construct a type of tight framelets using quadrature rules on M to represent the data (or a function) and to exploit the derived framelets to process the data (for example, image and signal processing on the sphere or graphs).

One critical computation for framelets is to compute, from the framelet coefficients for the input data (which are assumed at the highest level), the framelet coefficients at lower levels, and also to evaluate the function values at new nodes using the framelet representation. We design an efficient computational strategy, which we call fast framelet filter bank transform (FMT), to compute the framelet coefficients and to recover the function. Assuming the fast Fourier transform (FFT) and using polynomial-exact quadrature rules on the manifold M, the FMT has the same computational complexity as the FFT. Numerical examples illustrate the efficiency and accuracy of the algorithm for the framelets.