Minimal Green energy points

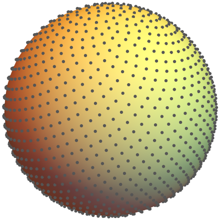
Let $\mathcal{M}$ be a compact Riemannian manifold and let $G(x, y)$ be the Green function for the Laplacian:

$$\Delta_x G(x, y) = \delta_y(x) - V^{-1}\text{vol}$$

Define the (discrete) Green energy by

$$E_G(x_1, ..., x_N) = \sum_{i \neq j} G(x_i, x_j).$$

If $\mathcal{M} = \mathbb{S}^2$, then $G(x, y) = \log \|x - y\|^{-1}$ (essentially).



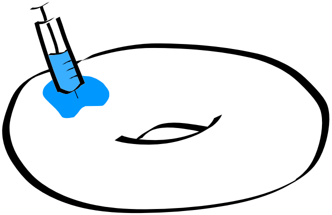
Separation distance

Theorem

For a collection of N minimal logarithmic energy points on $\mathbb{S}^2$ *there is a radius* $r = r(N)$ *such that* $x_i \notin B(x_j, r)$ *for every* $i \neq j$.

$\Rightarrow$ Separation distance result.

The natural "area of influence" in a general compact manifold appears to be not a geodesic ball, but a *harmonic ball*.



$B^{\text{harm}}(p, t) =$ the blob of t units of fluid injected at p.