

Packings of Superdisks

joint work with Maryna Viazovska

Optimal packing

Jiao, Stillinger, Torquato (2009)

Superdisk $B_2^p = \{(x, y) \in \mathbb{R}^2 : |x|^p + |y|^p \leq 1\}$

Linear programming bound

Cohn, Elkies 2003:

$$\delta^t(K) \leq \inf f(0)$$

$$f \in S(\mathbb{R}^n)$$

$$\hat{f}(0) \geq \text{vol } K$$

$$\hat{f}(u) \geq 0 \quad \forall u \in \mathbb{R}^n \setminus \{0\}$$

$$f(x) \leq 0 \quad \forall x \in K^\circ - K^\circ$$

$$\hat{f}(u) = p(u) e^{-\pi \|u\|^2}$$



$$\delta^t(K) \leq \inf \int_{\mathbb{R}^n} p(u) e^{-\pi \|u\|^2} du$$

$$p(u) \in \mathbb{R}[u]_{\leq 2d}$$

$$p(0) \geq \text{vol } K$$

$$p(u) \geq 0 \quad \forall u \in \mathbb{R}^n \setminus \{0\}$$

$$\int_{\mathbb{R}^n} p(u) e^{-\pi \|u\|^2} e^{2\pi i u \cdot x} du \leq 0 \quad \forall x \in K^\circ - K^\circ$$

Computational Experiments

P	optimal density	upper bound	max degree
2	0.9069001	0.9077117	38
4	0.9513761	0.9519221	36
6	0.9754212	0.9757366	36
8	0.9852472	0.9858322	34
10	0.9901719	0.9903665	34

Next steps:

- Verification
⇒ rigorous bounds
- Properties of solution?