- $A \subset \mathbb{R}^p$ compact, infinite; $\dim_H A = d$
- $\omega_N \subset A$ discrete set $\longleftrightarrow$ $\nu_N := \frac{1}{N}\sum_{\boldsymbol{x}\in\omega_N} \delta_{\boldsymbol{x}}$ – associated measure
- $E_s(\omega_N) := \sum_{\boldsymbol{x}\neq\boldsymbol{y}\in\omega_N} |\boldsymbol{x}-\boldsymbol{y}|^{-s}$ $s > d$ – Riesz s-energy
- for A d-rectifiable: **any** sequence of minimizers $\{\tilde{\omega}_N : N \geq 1\}$ has asymptotics

$$\lim_{N\to\infty} \frac{E_s(\tilde{\omega}_N)}{N^{1+s/d}} = g_{s,d}(A)$$

- Conversely, if $\{\omega_N : N \geq 1\}$ has the right asymptotics of $E_s(\omega_N)/N^{1+s/d}$, it converges weak* to the Hausdorff measure

$$\nu_N \xrightarrow{\;*\;} \frac{\mathcal{H}_d}{\mathcal{H}_d(A)}$$

- on a self-similar fractal – **no** asymptotics for **all** minimizers
- but for $\underline{\omega}_N$, $N \in \underline{\mathcal{N}}$ with

$$\lim_{\underline{\mathcal{N}} \ni N \to \infty} \frac{E_s(\underline{\omega}_N)}{N^{1+s/d}} = \liminf_{N \to \infty} \frac{E_s(\omega_N)}{N^{1+s/d}} =: \underline{g}_{s,d}(A)$$

still:

$$\underline{\nu}_N \xrightarrow{*} \frac{\mathcal{H}_d}{\mathcal{H}_d(A)}$$

- $\underline{\nu}$ also converges for rectifiable$\bigcup$fractal with separated union
- but apparently **breaks** for fractal$\bigcup$fractal even with separation
- motivation: packing distance does not change for $N \in [2^p + 1, 2^{p+1}]$

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