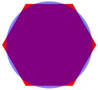
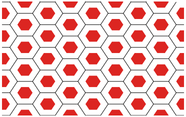
- The unit-distance graph $G(\mathbb{R}^n, \|\cdot\|)$: fix $\|\cdot\|$ on $\mathbb{R}^n$.
  Let $G = (V, E)$ with $V = \mathbb{R}^n$ and $\{x, y\} \in E \Leftrightarrow \|x - y\| = 1$.
  $A \subset \mathbb{R}^n$ is 1-avoiding if it is an independent set in G.
- Question: What is the supreme density $m_1(\mathbb{R}^n, \|\cdot\|)$ achieved by a 1-avoiding set in $\mathbb{R}^n$?
- Motivation: lower bound for the chromatic number of $\mathbb{R}^n$.
- The Euclidean plane:
  Best construction: $m_1(\mathbb{R}^2, \|\cdot\|_2) \geq 0.229$.
  Erdös Conjecture: $m_1(\mathbb{R}^2, \|\cdot\|_2) < 1/4$.
  Best upper bound: $m_1(\mathbb{R}^2, \|\cdot\|_2) < 0.258795$.

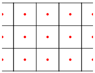
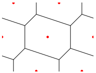


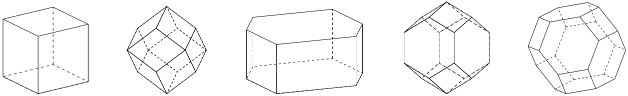
- Let $\|\cdot\|_{\mathcal{P}}$ such that the unit ball $\mathcal{P}$ tiles $\mathbb{R}^n$ by translation.

  Then $m_1(\mathbb{R}^n, \|\cdot\|_{\mathcal{P}}) \geq \frac{1}{2^n}$.



- Bachoc and Robins Conjecture: in this situation, $m_1(\mathbb{R}^n, \|\cdot\|_{\mathcal{P}}) = \frac{1}{2^n}$.

- Results:
  - n=2:  
  - Voronoi region of A_n.
  - Voronoi region of D_n: $m_1(\mathbb{R}^n, \|\cdot\|_{\mathcal{P}}) \leq \frac{1}{(3/4)2^n+n-1}$.
  - n=3:



$\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$ $\frac{1}{8}$???

These results are obtained with combinatorial methods, by solving packing problems in discrete subgraphs.

- Open Questions:
  - Settle dimension 3, improve the bound for D_n.
    Is there a standard induced subgraph leading to the $1/2^n$ bound?
  - Bounds coming from other methods, for instance involving Fourier transform of measures supported by the surface of the polytope.
    Asymptotic bounds?
  - What about other polytopes? (upper bounds and lower bounds)