Needlets, Designs and Fast Transforms on Manifolds

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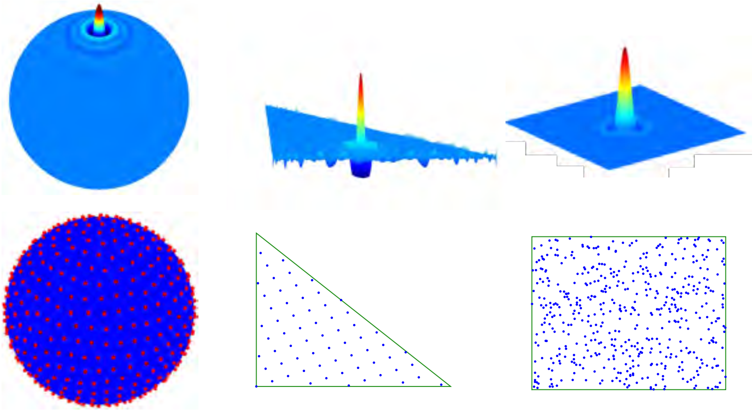
ICERM, 2018

Needlets and Designs on Manifolds

Riemannian Manifold $\mathcal{M}$: Δ $\{(\lambda_\ell^2, u_\ell)\}_{\ell=0}^{\infty}$

E.g. $\mathbb{S}^d$, T^d, $\mathcal{G}_{k,d}$, G

$$\psi_{j,k}(\boldsymbol{x}) := \sqrt{\omega_{j,k}} \sum_{\ell=0}^{\infty} h\left(\frac{\lambda_\ell}{2^j}\right) \overline{u_\ell(\boldsymbol{x}_{j,k})} u_\ell(\boldsymbol{x}), \quad \{(\omega_{j,k}, \boldsymbol{x}_{j,k})\}_{k=1}^{N_j}.$$

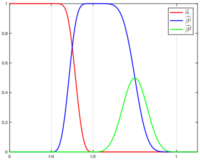


Fast Needlet Transforms (F$\mathcal{M}$T)

$$v_{j,k} := \langle f, \boldsymbol{\varphi}_{j,k} \rangle, \ \mathrm{v}_j := (v_{j,1}, \dots, v_{j,N_j}) \approx (f(\boldsymbol{x}_{j,1}), \dots, f(\boldsymbol{x}_{j,N_j}))$$

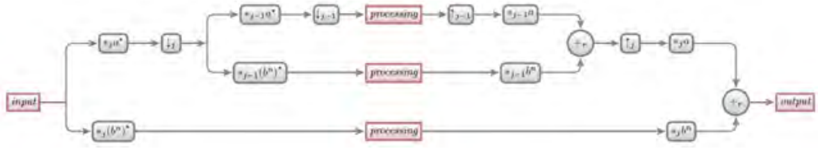
$$w^n_{j,k} := \left\langle f, \boldsymbol{\psi}^n_{j,k} \right\rangle, \ \mathrm{w}^n_j := (w^n_{j,1}, \dots, w^n_{j,N_j}), \quad n = 1, \dots, r.$$

$$\mathrm{v}_{j-1} = (\mathrm{v}_j *_j a^\star)\!\downarrow_j = \mathbf{F}_{j-1}(\mathrm{v}_j *_j a^\star), \quad \mathrm{w}^n_{j-1} = \mathrm{v}_j *_j b^\star_n.$$



$h \leftarrow \widehat{\alpha}, \widehat{\beta^1}, \widehat{\beta^2}$

FFT on $\mathcal{M}$ is $\mathcal{O}\left(N_j(\log(N_j))^m\right)$, so is F$\mathcal{M}$T.



- Fast generate points on $\mathcal{M}$

$$\int_{\mathcal{M}} P_{2^j}(\boldsymbol{x})\, \mathrm{d}\sigma_d(\boldsymbol{x}) = \sum_{k=1}^{N_j} \omega_{j,k} P_{2^j}(\boldsymbol{x}_{j,k}).$$

- FFTs ($\mathbb{S}^2$, T^2, $\mathcal{G}_{2,4}$, G)