Non-Convex Relaxations for Rank Regularization

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The problem of finding a low rank approximation of a given measurement matrix is of key interest in computer vision. If we have obtained measurements of all the matrix elements of then a closed form solution is available through factorization. However, in general when we measure linear combinations of the elements the problem becomes more difficult. A common approache is to replace the rank term with the weaker (but convex) nuclear norm. This option gives an easy optimizaiton problem but suffers from a shrinking bias that can severely degrade the solution in the presence of noise.

I this talk I will present an alternative class non-convex regularizations that do not suffer from the same bias since it does not penalize large singular values. Our main theoretical results show that if a RIP holds then the stationary points are often well separated, in the sense that their differences must be of high rank. Thus, with a suitable initial solution the approach is unlikely to fall into a bad local minimum.
Our numerical tests show that the approach is likely to converge to a better solution than standard nuclear-norm relaxation even when starting from trivial initializations. I will then show how to reformulate the non-differentiable problem formulation term into a non-linear least squares problem with differentiable residuals using a bilinear parameterization.
This opens up the possibility of using second order methods such as Levenberg-Marquardt (LM) and Variable Projection (VarPro) to achieve accurate solutions for ill-conditioned problems.