

On variational methods in imaging sciences, and Hamilton-Jacobi equations

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We consider standard finite-dimensional variational models used in signal/image processing that consist in minimizing an energy involving a data fidelity term and a regularization term. We propose

new remarks from a theoretical perspective which give a precise description on how the solutions of the optimization problem depend on the amount of smoothing effects and the data itself. The dependence of the minimal values of the energy is shown to be ruled by Hamilton-Jacobi equations, while the minimizers $u(x,t)$ for the observed images x and smoothing parameters t are given by $u(x,t) = x - \nabla H(\nabla E(x,t))$ where $E(x,t)$ is the minimal value of the energy and H is a Hamiltonian related to the data fidelity term.

Various vanishing smoothing parameter results are derived illustrating the role played by the prior in such limits. Finally, we briefly describe how optimization methods using in imaging sciences can be used to overcome the curse of dimensionality for certain Hamilton-Jacobi equations arising in control theory.