Embedding properties of data-driven dissipative reduced order Models

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Realizations of reduced order models of passive SISO or MIMO LTE problems can be transformed to tridiagonal and block-tridiagonal forms, respectively, via different modifications of the Lanczos algorithm. Generally, such realizations can be interpreted as ladder resistor-capacitor-inductor (RCL) networks. They gave rise to network syntheses in the first half of the 20th century that was at the base of modern electronics design and consecutively to MOR that tremendously impacted many areas of engineering (electrical, mechanical, aerospace, etc.) by enabling efficient compression of the underlining dynamical systems. In his seminal 1950s works Krein realized that in addition to their compressing properties, network realizations can be used to embed the data back into the state space of the underlying continuum problems.

In more recent works of the first author (with Liliana Borcea, Fernando Guevara Vasquez, David Ingerman, Leonid Knizhnerman, Alexander Mamonov, Shari Moscow and Michael Zaslavsky) Krein's ideas gave rise to so-called finite-difference Gaussian quadrature rules (FDGQR), allowing to approximately map the data-driven ROM state-space representation to its full order continuum counterpart on a judicially chosen grid. Thus, the state variables can be accessed directly from the transfer function without solving the full problem and even explicit knowledge of the PDE coefficients in the interior, i.e., the FDGQR directly ``learns"" the problem from its transfer function. This embedding property found applications in PDE solvers, inverse problems and machine learning.