Parametrized Partial Differential Equations and Back-of-the-Envelope Calculations
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Undergraduate engineering students — and ultimately practicing engineers — develop good approximations to very complicated problems by judicious application of a relatively small set of domain ingredients. These ""Back-of-the-Envelope'' (BE) calculations offer several advantages: the BE domain ingredients — physical properties, constitutive laws, analysis approaches, and reference solutions — are shared by a large community and hence reflect continual verification; a BE calculation is transparent and reproducible and hence admits assessment — and blunder detection — by any domain expert; a BE calculation is operationally simple and hence provides the rapid response required in conceptual design and optimization studies. Typically a BE calculation will serve as a complement to, and as an independent confirmation of, computational treatment of a high-fidelity model.

Back-of-the-Envelope calculation can be viewed as a classification process: a map from a problem statement to an instantiation of the ``closest'' problem class in the domain canon; in many engineering domains, a problem class is characterized by a parametrized partial differential equation and associated (approximate) solution procedure. In this talk we describe the many ways in which model reduction has played — and can play — an important role in BE calculations; we also highlight the many mathematical and computational issues which remain unresolved. Examples are drawn primarily from heat transfer, but with some discussion also of acoustics, vibration, and solid mechanics and structures.