A Reduced Order Modeling approach for PDEs with bifurcating solutions

Annalisa Quaini, University of Houston

We propose a computationally efficient framework to treat nonlinear partial differential equations having bifurcating solutions as one or more physical control parameters are varied. Our focus is on steady bifurcations. Plotting a bifurcation diagram entails computing multiple solutions of a parametrized, nonlinear problem, which can be extremely expensive in terms of computational time. In order to reduce these demanding computational costs, our approach combines a continuation technique and Newton's method with a Reduced Order Modeling (ROM) technique, suitably supplemented with a hyper-reduction method. To demonstrate the effectiveness of our ROM approach, we trace steady solution branches for Navier-Stokes problems and for a nonlinear Schroedinger equation, called Gross--Pitaevskii equation, as one or two physical parameters are varied.

As an improvement over the proposed method, we present a localized ROM approach. We use the k-means algorithm to cluster snapshots and construct local POD bases, one for each cluster. The localized method can detect which cluster a new parameter point belongs to. Then, the local basis corresponding to that cluster is used to determine a ROM approximation. Since the cluster selection for a new parameter of interest is crucial to the overall accuracy, we will discuss several options."