Randomization for the Efficient Reduced Order Models in Diffuse Optical Tomography
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Nonlinear inverse problems appear in many applications for identification and localization of anomalous regions, such as finding tumors in the body, luggage screening, and finding contaminant pools in the earth. In this work, we focus on diffuse optical tomography (DOT) in medical image reconstruction. In DOT, we aim to recover an unknown image of interest, such as cancerous tissue in a given medium, using a mathematical model (the forward model) combined with measurements. The forward model in DOT is a diffusion model for the photon flux. The main computational bottleneck in such inverse problems is the repeated evaluation of a large-scale forward model. For DOT, this corresponds to solving large linear systems for each source and frequency at each optimization step. In addition, as Newton methods are very effective for these problems, we need to solve linear systems with the adjoint for each detector and frequency at each optimization step to efficiently compute derivative information. As rapid advances in technology allow for large numbers of sources and detectors, these problems become computationally prohibitively expensive.

In the past, the use of reduced order models (ROM) has been proposed to drastically reduce the size of the linear systems solved in each optimization step in DOT, while still solving the inverse problem accurately. This approach significantly reduces the cost of the inversion process by drastically reducing the computational cost of solving the forward problems. However, interpolatory model reduction requires the solution of large linear systems for all sources and frequencies as well as for all detectors and frequencies for each interpolation point in parameter space, followed by an expensive rank-revealing factorization to reduce the dimension. Hence, as the number of sources and detectors increases, even the construction of the ROM bases still incurs a substantial cost in the offline stage.

Since candidate ROM bases are (nearly) low rank, we propose to employ randomization to capture essentially the same subspace at much lower cost. Recall that using ROMs drastically reduces the size of the linear systems solved in each step of the optimization and our randomization technique reduces the number of large linear system solves needed for the ROM basis, combining these two approaches results in an effective and computationally highly efficient approach for nonlinear parameter inversion. Furthermore, using randomization for the efficient computation of ROM can be applied to numerous large-scale optimization problems.

We also plan to provide a brief theoretical justification for exploiting low rank structure in the reduction basis. Then, we link our approach, using randomization to compute the interpolatory model reduction bases, to tangential interpolation.