Stochastic Reduced Models: L_2 projections and L_1 approximations

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I will describe two recent procedures for stochastic model reduction. The first approach, based on adapted isometries in Gaussian spaces, permits the development of highly accurate Polynomial Chaos expansions for specific quantities of interest (QoI). An optimal isometry is identified that localizes the norm of the QoI in a neighborhood of a low-dimensional (nonlinear) manifold. Projectors on that manifold are easily constructed by composition of the isometry with multivariate polynomials. In the second approach, the physical system is reduced to a joint pdf for a subset of its observables. The choice of observables is paramount and will in general yield different reduced models. The data points associated with these observables are first abstracted as a diffusion process evolving on the graph described by the data. An embedding from the ambient space to the dominant eigenspace of the diffusion matrix localizes the data in a convenient manner that can be attained by projection. A projected Ito equation is then developed that takes advantage of this construction, generating samples of the initial graph that are constrained to the same manifold as the training set while sharing its salient statistical features.

The two approaches will be demonstrated on problems of interest in science and engineering.