Extreme event probability estimation using large-deviation theory and PDE-constrained optimization, with application to tsunamis

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We propose methods for the estimation of extreme event probabilities in complex systems governed by PDEs. Our approach is guided by ideas from large deviation theory (LDT) and borrows methods from PDE-constrained optimization. The systems under consideration involve random parameters and we are interested in quantifying the probability that a scalar function of the system state solution is at or above a threshold. Our methods first compute parameters that minimize the LDT-rate function over the set of parameters leading to extreme events. These solutions provide asymptotic information about small probability events. We propose several methods to refine these estimates, namely methods based on importance sampling and on geometric approximation of the extreme event sets. Theoretical and numerical arguments show that the performance of our methods is insensitive to the extremeness of the event. We illustrate the application of our approach to quantify the probability of extreme tsunami events on shore. Tsunamis are typically caused by a sudden, unpredictable change of the ocean floor elevation during an earthquake. We model this change as random process and use the one-dimensional shallow water equation to model tsunamis. We present a comparison of the methods for extreme event probability estimation, and find which type of ocean floor elevation change leads to the largest tsunamis on shore. This is joint work with Shanyin Tong and Eric Vanden-Eijnden from NYU.