Spatiotemporal pattern extraction by spectral analysis of vector-valued observables

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We present a data-driven framework for extracting spatiotemporal patterns generated by ergodic dynamical systems. Our approach is based on an eigendecomposition of a kernel integral operator acting on a Hilbert space of vector-valued observables of the system, taking values in a space of functions (scalar fields) on a spatial domain. This operator is constructed by combining aspects of the theory of operator-valued kernels for multitask machine learning with delay-coordinate maps of dynamical systems. Specifically, delay-coordinate maps performed pointwise in the spatial domain induce an operator acting on functions on that domain for each pair of dynamical states, whose eigenfunctions can recover spatiotemporal patterns with a non-separable structure in the temporal and spatial coordinates. We discuss two properties of this class of kernel integral operators, namely that they have, in the limit of infinitely many delays, common eigenspaces with the Koopman operator governing the evolution of vector-valued observables under the dynamics, and that they naturally quotient out dynamical symmetries. We present applications of this framework to the Kuramoto-Sivashinsky model, which demonstrate considerable performance gains in efficient and meaningful decomposition over eigendecomposition techniques utilizing scalar-valued kernels.