Complexity of 4D Pascal Determinant (and Permanent)

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Let $\rho(i_1, i_2, i_3, i_4); 1 \leq i_1, i_2, i_3, i_4 \leq n$ be 4D tensor. Its Pascal Determinant is defined as $PD(\rho) =: \sum_{\tau_2,\tau_3,\tau_4 \in S_n} (-1)^{sign(\tau_2\tau_3\tau_4)} \prod_{1 \leq i \leq n} rho(i, \tau_2(i), \tau_3(i), \tau_4(i))$, its Pascal Permanent is defined as
$PPe(\rho) =: \sum_{\tau_2,\tau_3,\tau_4 \in S_n} \prod_{1 \leq i \leq n} rho(i, \tau_2(i), \tau_3(i), \tau_4(i))$,

1. I will explain that 4D Pascal Determinant and Permanent are both VNP-complete.

2. It is possible that 4D Pascal Determinant is "harder" than 4D Pascal Permanent. It will be explained that 4D Pascal Permanent can be computed in $8^n poly(n)$ ops whereas 4D Pascal Determinant in $n!poly(n)$. Those two observations follow from the inequalities on the Waring Rank (WR) of the determinant and the permanent: $WR(Per_n) \approx 4^n, 4^n \leq\approx WR(Der_n) \leq (n+1)!$.

3. A 4D tensor $\rho(i_1, i_2, i_3, i_4); 1 \leq i_1, i_2, i_3, i_4 \leq n$ is called PSD (denoted as $\rho \succeq 0$) if the $n^2 \times n^2$ matrix $\rho(i_1, i_2; i_3, i_4)$ is positive semidefinite. In the PSD case, both $PD(\rho), PPe(\rho) \geq 0$. I will show that deciding whether $PPe(\rho) = 0$ is NP-HARD for PSD tensors; deciding whether $PD(\rho) = 0$ is equivalent to testing whether a symbolic determinant is zero, and thus is in BPP. Finally, I will present a randomized $4^n poly(\epsilon^{-1})$ algorithm to approximate $PD(\rho), \rho \succeq 0$ with the relative factor $(1 + \epsilon)$.