

KP solitons and cluster algebras

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We start with a realization of the totally non-negative (tnn) part of the Grassmannian $Gr(N,M)$ in terms of the soliton solutions of the KP equation. We then construct a decomposition of the tnn part of the Grassmannian according to the "asymptotic" spatial pattern of the soliton solutions. This leads to a classification theorem of all soliton solutions of the KP equation, showing that each soliton solution is uniquely parametrized by a derangement of the symmetric group S_M . Each derangement defines the so-called Le-diagram of Postnikov. Then we show that the Le-diagram provides a complete classification of the "entire" spatial patterns of the soliton solutions coming from the tnn part of $Gr(N,M)$ for asymptotic values of the time. We also classify the spatial patterns of the soliton solutions from the totally positive part of $Gr(2,M)$, and give a soliton interpretation of the cluster algebra structure of A-type. In particular, we construct the associahedron in the space of multi-times of the KP hierarchy where each time represents the flow parameter of symmetry of the KP equation. This talk is based on a joint work with Lauren Williams.