A high-order positivity preserving method for the Vlasov-Poisson

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The Vlasov equation is a collisionless Boltzmann equation that is effective for describing a plasma far from thermodynamic equilibrium. Electromagnetic fields produce complicated structures that full phase-space simulations can capture, whereas moment based methods struggle with describing. Popular methods for discretizing phase space include the so-called Particle in Cell (PIC) method and mesh-based Eulerian methods.

One of the chief numerical difficulties in solving the Vlasov-Poisson system is the time step restriction attributed to moderate to large velocities in the system. Explicit Eulerian methods require excessively small time steps, and implicit Eulerian methods have matrices with large condition numbers. Particle in Cell methods together with their Lagrangian and semi-Lagrangian counterparts relax this time step restriction by solving for characteristics of the system. Operator splitting reduces the complication of these characteristics and is common practice for PIC and semi-Lagrangian methods.

In this work, we demonstrate the benefits of using high-order methods for solving the Vlasov-Poisson equations. For the same amount of computational effort and storage, high-order methods capture the solution more accurately than lower-order methods. We present a novel hybrid semi-Lagrangian discontinuous Galerkin method that uses an unstructured grid for configuration space and a structured Cartesian grid for velocity space. The structured grid that allows for genuinely high-order semi-Lagrangian time-stepping to be performed, and the unstructured grid allows for complex geometries and boundary conditions to be accommodated. We present 1D-1V results as well as 2D-2V results including the formation of a plasma sheath in the proximity of a cylindrical Langmuir probe.

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