An Efficient Discrete Velocity Method for the Boltzmann Equation

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A discrete velocity scheme for solving the Boltzmann equation will be described and demonstrated by application to homogeneous and one-dimensional problems. Direct solution of the Boltzmann equation is computationally expensive because, in addition to working in physical space, the nonlinear collision integral must also be evaluated in velocity space. In principle, collisions between each point in velocity space with all other points in velocity space must be considered when evaluating the collision integral, but this is very expensive. Motivated by the Direct Simulation Monte Carlo (DSMC) method, we reduce the computational cost in the present method by randomly sampling a set of collision partners at each point in velocity space. Unlike some other Monte Carlo methods for evaluation of the collision integral, mass, momentum and energy are conserved exactly in our scheme. The method is tested by solving several homogeneous relaxation problems, and is applied to traveling 1D shock waves. Since a coarse velocity discretization is required for efficient calculation, the effects of different velocity grid resolutions are examined. When the performance is compared to DSMC and it is found that the new scheme performed nearly an order of magnitude faster than DSMC upstream of the shock wave for the same noise level. Downstream of the shock wave the noise levels are comparable for the same computational effort. The extension of the method to gases with internal energy and gas mixtures will also be described.