Fast Spectral-Galerkin Methods for High-Dimensional PDEs in Unbounded Domains

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The talk will consist of two parts.

In the first part, I shall present spectral sparse grid methods for elliptic type equations in high dimensional unbounded domains. By using modified mapped Chebyshev functions as basis functions, we construct mapped Chebyshev sparse grid (MCSG) methods which enjoy the following properties: (i) the mapped Chebyshev approach enables us to build sparse grids with Smolyak's algorithms based on nested, spectrally accurate quadratures, and allows us to build fast transforms between the values at the sparse grid and the corresponding expansion coefficients; (ii) the mapped Chebyshev basis functions lead to identity mass matrices and very sparse stiffness matrices for problems with constant coefficients, and allow us to construct a matrix-vector product algorithm with quasi-optimal computational cost even for problems with variable coefficients; and (iii) the resultant linear systems for elliptic equations with constant or variable coefficients can be solved efficiently by using a suitable iterative scheme. As an application, we shall use the proposed MCSG method to solve the N-particle electronic Schrodinger equation.

In the second part, I shall present some preliminary work on constructing efficient Galerkin discretization for the collision operator using the multi wavelets.