High order asymptotic preserving methods for the Boltzmann equation

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The numerical solution of Boltzmann-type equations close to fluid regimes represents a real challenge for numerical methods. In these regimes, in fact, the intermolecular collision rate grows exponentially and the collisional time becomes very small. On the other hand, the actual time scale for evolution is the fluid dynamic time scale, which can be much larger than the collisional time. A non dimensional measure of the importance of collision is given by the Knudsen number which is large in the rarefied regions and small in the fluid ones. Standard computational approaches lose their efficiency due to the necessity of using very small time steps in deterministic schemes or, equivalently, a large number of collisions in probabilistic approaches.

Unfortunately the use of implicit solvers originates a prohibitive computational cost due to the high dimensionality and the nonlinearity of the collision operator. The possible approaches that permit to overcome such a difficulty can be subdivided into two main classes. Domain decomposition strategies and asymptotic preserving schemes. The first class of methods permits to avoid the problem of very small Knudsen number by identifying the regions where it is possible to use the reduced fluid model and the regions where the full kinetic model must be solved. A closely related research approach combines stochastic and deterministic solvers in the different regions by originating hybrid methods.

Concerning the asymptotic preserving strategies, these techniques aim at solving the full problem in the entire domain for all choices of time steps and Knudsen numbers. In this talk we discuss different numerical approaches recently developed which permit to integrate efficiently the Boltzmann equation in stiff regimes by achieving uniform high-order accuracy in time for a large range of Knudsen numbers and by avoiding the expensive implicit resolution of the collision operator. These include splitting methods, Runge-Kutta methods, exponential methods and multistep methods.

References

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