High order semi-Lagrangian schemes and operator splitting for the Boltzmann equation.

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A natural way of time-stepping the Boltzmann equation is operator splitting, where successive solutions of the ballistic and collision operators are composed in time and interleaved with appropriate field updates. This is the method of choice for particle codes, but it is also popular among mesh-based (discrete velocity) schemes, as it enables one to use semi-Lagrangian formulations of the ballistic operator, which are not subject to the Courant-Friedrichs-Lewy (CFL) stability limit that plagues explicit Eulerian methods.

When no mean force is present, operator splitting effectively decouples the configuration variables (x,y,z) from the velocity variables (vx,vy,vz); this results in only half the memory requirement of unsplit Runge-Kutta methods, because there is no need to store an intermediate stage solution for the full phase-space. In the general case with non-zero force, the result above is achieved by dimensional splitting of the ballistic operator, which is thus reduced to a series of constant advection steps. This 'fully-split' time-stepping algorithm relies on an accurate semi-Lagrangian 1D constant advection solver as the main building block.

We describe a high order, conservative and positivity preserving solution of the Boltzmann equation based on a cell-centered version of the Convected Scheme (CS), which extends previous analysis [1] to arbitrarily high order of accuracy in space; specifically, we discuss a spectrally accurate CS based on filtered fast Fourier transforms, and we apply it to the solution of the Vlasov-Poisson system on periodic domains. We show the scheme's behavior in typical 1D-1V test cases: linear and non-linear Landau damping, two-stream and bump-on-tail instabilities. Then we discuss higher dimensional problems, where the computational savings inherent to the spectral CS would enable unprecedented phase-space resolution. Finally we consider the solution of the Vlasov–Maxwell system, as well as the inclusion of collisional processes.

[1] Y. Güçlü and W.N.G. Hitchon. "A high order cell-centered semi-Lagrangian scheme for multi-dimensional kinetic simulations of neutral gas flows". Journal of Computational Physics, 2012, no. 231, pp. 3289–3316.