Multiscale asymptotic solutions to the Boltzmann equation for aerospace applications

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Abstract

Hydrodynamic models allows for simulation of atmospheric entry flows at altitudes where the continuum assumption is satisfied. Transport phenomena in atmospheric entry plasmas are characterized by strong nonequilibrium conditions resulting from distinct time scales for excitation and relaxation of the translational and internal energy modes of the gas particles, as well as for chemical reactions. Many engineering models were developed based on experimental data obtained in flight and in high-enthalpy facilities representative of specific flight conditions. These models contain ad-hoc terms only valid for some dedicated applications, whereas models derived more rigorously from kinetic theory often rely on assumptions too strong to accurately describe complex physico-chemical phenomena. This work is part of a broader effort that aims at developing new models based on microscopic theory and applying them to macroscopic scale in computational fluid dynamics codes. The general objectives are to enrich mathematical models by incorporating more physics in a rigorous way, to identify the mathematical structure of the equations derived, and to allow for integration of quantum chemistry databases. We propose to derive asymptotic solutions to multiscale kinetic equations with entangled collision operators characteristic of atmospheric entry plasmas. Assuming that the Knudsen number is small enough, a multiscale Chapman-Enskog expansion method is used by accounting for the electromagnetic field influence, as well as for a possible thermal nonequilibrium of the translational energy mode of the electrons and heavy particles, such as atoms, molecules, and ions, given their strong disparity of mass. Then, reactive collisions are considered in a mixture of atomic gases. Finally, the relaxation of internal energy is studied in a molecular gas. The scaling used in the multiscale asymptotic treatment is derived from a dimensional analysis of the Boltzmann equation. The collisional invariants are identified in the kernel of the collision operators and macroscopic conservation equations follow from Fredholm's alternative. The macroscopic equations derived satisfy the laws of thermodynamics and the law of mass action. Well-posedness of the transport properties is established, provided that some conditions on the kinetic data are met. The mathematical structure of the transport matrices is readily used to build transport algorithms (direct linear solver / convergent iterative Krylov projection methods). The present study also yields a rigorous derivation of a set of macroscopic equations in the situation where hyperbolic and parabolic kinetic scalings are mixed.