A 3D hp Finite Element Code for Multiphysics Coupled Problems with Elements of All Shapes,Orientation Embedded Shape Functions, the Discontinuous Petrov Galerkin (DPG) Method and Other Stories

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I will start with a short overview of an ongoing work on a general three-dimensional hp code for hybrid meshes consisting of elements of ``all shapes'' (hexas, tets, prisms and pyramids), supporting H1, H(curl), H(div) and L2 conforming discretizations of coupled multiphysics problems and ANISOTROPIC hp refinements.

The main part of my talk is going to be quite technical and it will focus on the construction of ``orientation embedded'' shape functions for the elements of all shapes and the whole exact sequence. The work extends the idea from [1] and builds upon the fundamental results of Schoeberl, Zaglmayr, Beuchler, Pillwein, Nigam, Phillips, Bergot, Durufle, see e.g. [2,3,4,5] and the literature therein.

The exact sequence elements provide then a natural framework for coding various versions of the DPG method and standard model problems: diffusion-convection-reaction, elasticity, Maxwell and linear acoustics, Stokes etc. I will finish the talk by discussing such an implementation and flashing a few numerical examples.

[1] P. Gatto and L. Demkowicz, ``Construction of H^1-conforming hierarchical shape functions for
elements of all shapes and transfinite interpolation'', Finite Elements in Analysis and Design,
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[2] J. Schoeberl and S. Zaglmayr, ``High order Nedelec elements with local complete sequence
property'', International Journal for Computation and Mathematics in Electrical and Electronic
Engineering, 24(2):374-384, 2005.

[3] S. Beuchler, V. Pillwein and S. Zaglmayr, ``Sparsity optimized high order finite element
functions for H(curl) on tetrahedra'', Adv. Appl. Math., 50(5): 749-769.

[4] N. Nigam and J. Phillips, ``High-order conforming finite elements on pyramids'', IMA Journal of Numerical Analysis, 32(2):448-483, 2012.

[5] M. Bergot and M. Durufle, ``Approximation of H(div) with High-Order Optimal Finite Elements for
Pyramids, Prisms and Hexahedra'', Commun. Comput. Phys., 14(5): 1372-1414.