Advances in the mathematical theory of the finite element immersed boundary method

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The Immersed Boundary Method (IBM) is an effective mathematical model and approximation scheme for the discretization of biological systems which involve the interaction of fluids and solids.

The Finite Element IBM (FEIBM) proved to be competitive with respect to the original IBM (based on finite differences and on a suitable approximation of a Dirac delta function) in several aspects: in particular, the position of the solid can be dealt with in a natural way by taking advantage of the underlying variational formulation (thus avoiding the use of the delta function); moreover, the use of finite elements allows for sharp pressure jumps when discontinuous pressure schemes are adopted.

Recently, a fully variational approach of the FEIBM has been introduced, which can be shown to be unconditionally stable with respect to the time discretization. The novelty consists in the treatment of the coupling between the solid and the fluid: in the standard formulation, this is given by a differential equation stating that the velocity of the solid is equal to that of the fluid, while in the new formulation this coupling is imposed in a weak form. A rigorous mathematical analysis shows the stability of the coupling and the unconditional time stability.