Commuting projections on graphs
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Motivated by the increasing importance of large-scale networks typically modeled by graphs we study properties associated with the popular graph Laplacian. We exploit its mixed formulation based on its natural factorization as product of two operators. The goal is to construct a coarse version of the mixed form for the graph Laplacian operator with the purpose to construct a multilevel hierarchy of graphs and associated operators. In many situations in practice having a coarse (i.e., reduced dimension) model that maintains some inherent features of the original large-scale graph and respective graph Laplacian offers potential to develop efficient algorithms for analyzing the underlined network modeled by this large-scale graph. One possible application of such a hierarchy is to develop multilevel methods that have the potential to be of optimal complexity. We show how one can construct a projection from the "edge-space" onto a properly constructed coarse edge-space associated with the edges of a coarser graph. This projection commutes with the discrete divergence operator, and the pair of coarse edge-space and coarse vertex-space offer the potential to construct two-level, and by recursion, multilevel methods for the mixed formulation of the graph Laplacian. The construction is close in spirit to the classical pair of spaces Raviart-Thomas/piece-wise constants used to discretize the mixed form of Laplace equation. The performance of a two-level method with overlapping Schwarz smoothing and correction based on the constructed coarse spaces is illustrated on couple of examples. The presentation is based on ongoing work with Panayot Vassilevski.