**New Mixed Elements for Linear Elasticity Problems within the Hellinger-Reissner Variational Formulation**

Jun Hu, Peking University

A family of rectangular mixed elements are proposed for solving the first order system of linear elasticity equations in any space dimension, where the stress field is approximated by symmetric finite element tensors. This family of elements has a perfect matching between the stress components and the displacement. The discrete spaces for the normal stress $\sigma_{ii}$, the shear stress $\sigma_{ij}$ and the displacement $u_i$ are $\operatorname{span}\{1,x_i\}$, $\operatorname{span}\{1,x_i,x_j\}$ and $\operatorname{span}\{1\}$, respectively, on rectangular grids. In particular, the definition remains the same for all space dimensions. As a result of these choices, the theoretical analysis is independent of the spatial dimension as well. In 1D, this element is nothing else but the 1D Raviart-Thomas element, which is the only conforming element in this family. In 2D and higher dimensions, there are new elements but of the minimal degrees of freedom. The total degrees of freedom per element is $2$ plus $1$ in 1D, 7 plus 2 in 2D, and 15 plus 3 in 3D. The previous record of the least degrees of freedom is, 13 plus 4 in 2D, and 54 plus 12 in 3D, on the rectangular grid. These elements are the simplest element for any space dimension.

The above family of nonconforming mixed finite elements is enriched to result in a lower-order conforming mixed finite element method for linear elasticity problems in any space dimension. In the new conforming mixed method, the normal stresses are approximated by quadratic polynomials $\{1, x_i, x_i^2\}$, the shear stresses by bilinear polynomials $\{1, x_i, x_j, x_ix_j\}$, and the displacements by linear polynomials $\{1, x_i \}$. The number of total degrees of freedom per element is 10 plus 4 in 2D, and 21 plus 6 in 3D, while the previous record of least dof for conforming element is 17 plus 4 in 2D, and 72 plus 12 in 3D.

In two and three dimensions, this family of conforming rectangular mixed finite elements is extended to any order. For two dimensions, the normal stress of the matrix-valued stress field is approximated by an enriched Brezzi--Douglas--Fortin--Marini element of order $k$, and the shear stress by the serendipity element of order $k$, the displacement field by an enriched discontinuous vector-valued $P_{k-1}$ element. For three dimensions, the normal stress is approximated by an enriched Raviart--Thomas element of order $k$, and each component of the shear stress by a product space of the serendipity element space of two variables and the space of polynomials of degree $\leq k-1$ with respect to the rest variable, the displacement field by an enriched discontinuous vector-valued $Q_{k-1}$ element. A family of reduced elements is also proposed by dropping some interior bubble functions of the stress and employing the discontinuous vector-valued $P_{k-1}$ (resp. $Q_{k-1}$) element for the displacement field on each element. As a result the lowest order elements have $8$ plus $2$ and $18$ plus $3$ degrees of freedom on each element for two and three dimensions, respectively. Finally, a family of conforming triangular mixed elements is proposed.