A brief survey on tensor rank and tensor decomposition, from a geometric perspective.

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The rank of a tensor is the minimum number of summands needed to express it as a sum of decomposable tensor. In the case of matrices, this definition coincides with matrix rank. Such a minimal sum is a tensor decomposition, called Candecomp/Parafac, or Waring decomposition in the symmetric case.

We show some analogies and differences of tensor rank with matrix rank, touching the spectral theory of tensors, the variational approach by Lim and Qi, and the hyperdeterminant. Singular vector dples generalize singular vector pairs of a matrix, but their behaviour is more elusive, for example they give the best rank one approximation but do not help to find higher rank approximations.
We consider the geometry of tensor spaces, involving the secant varieties of Segre and Veronese varieties. We discuss their properties and some basic questions which are still open, although a pattern is emerging from the research of last years.

We expose a technique, developed jointly with Landsberg and Oeding, to actually decompose a tensor of small rank, by using tensor eigenvectors, with some examples.