On a Real Variant of Smale's 17th Problem

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While there are numerous deterministic hardness results on solving polynomial systems over the real and complex numbers, the power of randomization has yet to be fully understood.

Recent advances by Beltran, Pardo, Burgisser, and Cucker have shown that, on average, one can approximate single complex roots in polynomial-time. (Technically, these results give an almost complete solution to Smale's 17th Problem.) Over the reals, the average-case complexity of numerical solving is more subtle, since the number of roots is no longer constant with probability 1. This forms part of a real analogue of Smale's 17th Problem, formulated by the author and Yinyu Ye around 2005.

We show how to count real roots, with high probability, in polynomial-time under certain conditions. Our algorithms are based on A-discriminants and Archimedean tropical varieties. Unlike other recent methods, our algorithms run in time polynomial in the logarithm of the degree of the underlying polynomials.