Recovery of algebraic-exponential data from moments and a generalization of the Lowner-John ellipsoid problem
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Let $G$ be a bounded open subset of Euclidean space with real algebraic boundary $\Gamma$. In a first part of the talk we consider the case where $G=\{x: g(x) \leq 1\}$ for some quasi-homogeneous polynomial $g$ and derive several properties of $G$ as well as the non-Gaussian integral $\int \exp(-g)dx$. In particular, we show that the volume of $G$ is a convex function of the coefficients of $g$ and solve a generalization of the Lowner-John problem.

Next, we consider a more general case and under the assumption that the degree $d$ of $\Gamma$ is given, and the power moments of the Lebesgue measure on $G$ are known up to order $3d$, we describe an algorithmic procedure for obtaining a polynomial vanishing on $\Gamma$. The particular case of semi-algebraic sets defined by a single polynomial inequality raises an intriguing question related to the finite determinateness of the full moment sequence. The more general case of a measure with density equal to the exponential of a polynomial is treated in parallel. Our approach relies on Stokes theorem and simple Hankel-type matrix identities.