R-operator and knot invariants via cluster algebra

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The cluster algebra was introduced by Fomin and Zelevinsky around 2000. A characteristic operation of the algebra called `mutation' has been related to various notions in mathematics. In this talk, based on joint work with Kazuhiro Hikami (Kyushu University), I introduce our recent research on the R-operator constructed from the mutations.

First we define the R-operator and apply it to study the complex volume, (hyperbolic volume) + i (Chern-Simons invariant), of knot complements in S^3. In three-dimensional hyperbolic geometry a mutation produces an ideal tetrahedron, and the R-operator corresponds to a hyperbolic octahedron. We show that the cluster variables are interpreted as Zickert's edge parameters used to compute complex volumes. Second we introduce the q-deformation of the R-operator by using quantum cluster algebra a la Fock and Goncharov. We construct a braiding operator in terms of quantum dilogarithm function, which realizes the quantum R-operator as adjoint action. In a limit that q goes to a root of unity, the braiding operator reduces to the Kashaev R-matrix up to a simple gauge-transformation.

References: arXiv:1304.4776, arXiv:1404.2009.