

Ap\ 'ery numbers and their experimental siblings

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The Ap\ 'ery numbers are the famous sequence which underlies Ap\ 'ery's proof of the irrationality of $\zeta(3)$. Following a question of Beukers, a computer search of Zagier has revealed a finite collection of related sequences which occur as integral solutions of Ap\ 'ery-like recurrence equations. This search has since been extended to more general recurrence equations, and in each case only finitely many sequences appear to exist. Among the remarkable properties of these sequences are connections with modular forms and so-called supercongruences, some of which remain conjectural. We review these properties and demonstrate how Ap\ 'ery-like numbers and their connection with modular forms feature in various, apparently unrelated, problems, such as the theory of short random walks or the computation of digits of π .