Computational challenges in experimental mathematics

David H. Bailey, Lawrence Berkeley National Laboratory and UC Davis

Very high-precision arithmetic (up to as high as 50,000 to 100,000 digits) is emerging as an increasingly essential tool for experimental mathematics. When combined with integer relation algorithms such as the PSLQ algorithm, very high-precision computing facilities permit one to tentatively recognize computed values of integrals, series, limits or other values, in terms of compact, analytic formulas. We illustrate this methodology with a number of examples, including the discovery of a new formula for pi, the resolution of three families of definite integrals that have roots in quantum field theory, the analysis of Poisson lattice sums, and the analysis of Mordell-Tornheim-Witten sums. We then summarize lessons learned from these studies. These include the pressing need to develop faster algorithms for numerical multi-dimensional integration, special function evaluation and integer relation detection, and the efficient implementation of such software on highly parallel computer systems.