Computational complexity of problems in 3-dimensional topology

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Almost all computation questions about high-dimensional manifolds (e.g. deciding if a manifold is S^n) are undecidable. In contrast, in 3-dimensions many natural questions are algorithmic solvable, including whether two given manifolds are homeomorphic, though often these algorithms are based on deep theories (Geometrization, Floer homology). The question then becomes how hard is it to solve these questions, both in theory and in practice.

I will survey these issues of computational complexity, focusing on the question of computing the genus of a knot and its special case of deciding if a knot is knotted, including the foundational work of Haken from the 1960s as well as more recent work of Agol-Hass-Thurston, Kuperberg, Lackenby, Dynnikov, Dunfield-Hirani, and Dunfield-Friedl-Jackson, from both a theoretical an practical perspective. I will conclude with some examples of trying to compute the genus of random knots with more than one hundred crossings.

No prior knowledge of issues of computational complexity will be assumed.