

Minicourse - Nielsen-Thurston theory

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This minicourse introduces Nielsen-Thurston theory, our foundation for understanding mapping classes. Mapping classes are homeomorphisms of a surface, considered up to isotopy. These form a group under composition, again up to isotopy. (When the surface is an n -punctured disk, this group is the same as the braid group on n strands.) Nielsen-Thurston theory classifies mapping classes into one of three types -- periodic, reducible, and pseudo-Anosov -- which are more than analogous to the elliptic, parabolic, and hyperbolic elements of $SL(2, \mathbb{Z})$. In effect, this trichotomy reduces understanding mapping class behavior to understanding the pseudo-Anosov case. The theory associates to a pseudo-Anosov mapping class an invariant structure on the surface, which may be represented metrically by a rectangle decomposition, dynamically by a pair of measured foliations, or combinatorially by a train track. We will illustrate these ideas, with emphasis on computation and examples.