

Realizing Collinear Sequences with the Three-Body Problem

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The configurations space for the planar three-body problem has homotopically nontrivial loops (free homotopy classes) when we forbid collisions. The free homotopy class of a loop is encoded primarily by its syzygy sequence: the list of collinearities suffered as the three bodies move, written down in the order that they are suffered. If we pretend that the three-body configuration space is a compact Riemannian manifold and that its dynamics is geodesic flow, then every free homotopy class would be realized by a solution to the three-body problem. Is that really true? Yes! if the total angular momentum is very small but nonzero, and if the three masses are nearly equal then a recent theorem asserts that every free homotopy class is realized by a solution. But the answer may be "No!" when the total angular momentum is zero. We sketch ideas behind the proof of "yes", ideas developed by Rick Moeckel in the 80s. We sketch some rationale behind "no". Depending on audience and mood, I may focus more on the zero angular momentum problem than the "yes" answer. At a certain point in all this work, Mark Levi's mechanical intuition came to the rescue and I do ask my audience the favor of making sure I cover this point.