The evolution of spatial critical points under abrasion

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The best known mathematical models for the abrasion of sedimentary particles are curvature-driven flows [1]: a special class of nonlinear partial differential equations defining the evolution of a surface Σ by the speed v in the direction of its surface normal, and v is given as a function of the principal curvatures κ, λ of Σ:

(1) v=v(\kappa, \lambda)

While locally defined, curvature-driven flows have startling global properties, e.g. they can shrink curves and surfaces to round points. These features made these flows powerful tools to prove topological theorems which ultimately led, via their generalizations by Hamilton to Perelmans's celebrated proof of the Poincar\'e conjecture.

As a by-product of these great efforts, in 1987 Grayson [2] proved that if Σ is given as a distance function from a fixed reference O then the number $N(t)$ of spatial critical points (extrema of the distance) is decreasing monotonically under the planar $v=\kappa$ flow, also called the curve shortening flow.

We will show that there is mounting evidence that similar, though weaker (generic, stochastic) statements are true for static balance points on 3D solids evolving under (1). We rely on some results on curvature-driven flows [2] and also on results about stochastic models adopted in digital image processing to track the number of critical points [3][4][5].

We will also review historically the mathematical models of abrasion.

References:

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