Model order reduction for the numerical solution of diffusive inverse problems.

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We present a general framework for the numerical solution of ill-posed inverse problems with boundary data for the coefficients of diffusive partial differential equations (PDEs). This includes elliptic equations of the electrical impedance tomography (EIT) problem with Dirichlet-to-Neumann (DtN) map data and parabolic equations of the controlled source electromagnetic (CSEM) problem with the time dependent DtN map measurements. The framework is built around the construction of a reduced order model (ROM) of the spatial elliptic PDE operator directly from the measured data. For the 2D EIT problem the ROM is a resistor network with a critical circular planar graph. In the parabolic case the ROM is a projection of the spatial operator on a carefully chosen rational Krylov subspace.

The use of ROMs in inversion is twofold. First, in certain cases the coefficients of the ROMs can be interpreted as averages of the unknown PDE coefficient on the so-called optimal grid. Those can be used to obtain the reconstructions with a direct, non-iterative procedure. Second, one may construct a misfit objective functional from the ROM coefficients to improve the conventional optimization methods. These nonlinearly preconditioned functionals are much easier to minimize compared to the conventional data misfit functionals. In fact, in many practical cases a single Gauss-Newton iteration is sufficient to obtain high quality reconstructions. We illustrate both examples with extensive numerical results.

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