Stability of the interior problem of tomography and the spectral properties of the Finite Hilbert transform.

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We study the interior problem of tomography by using the Gelfand-Graev formula, which converts the tomographic data into the finite Hilbert transform (FHT) of an unknown function f along a collection of lines. Pick one such line, call it the x-axis, and assume that the function to be reconstructed depends on a one-dimensional argument by restricting f to the x-axis. Let I be the interval where f is supported, and J be the interval where the Hilbert transform of f can be computed using the Gelfand-Graev formula. The equation to be solved is Hf=g$, where H is the FHT that integrates over I and gives the result on J, i.e. H: L^2(I) -> L^2(J), and g is known on J. In the case of complete data, I is a subset of J, and the classical FHT inversion formula reconstructs f in a stable fashion. In the case of interior problem (i.e., when the tomographic data are truncated), I is no longer a subset of J, and the inversion problems becomes severely unstable. By using a differential operator L that commutes with H we determine the spectral properties of H depending on the relative positions of the intervals I and J.